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Transport Efficiency of Spaceplanes with Airbreathing Phases

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At present, many concepts of advanced space transportation systems are under discussion, and challenging technology programs are on the way. Vehicles relying on airbreathing propulsion seem particularly promising. In this paper, simplified solutions of the equations of motion are developed and evaluated to compare the transport efficiency of three space vehicle concepts (single-stage-to-orbit and two-stage-to-orbit) that combine airbreathing and rocket-propelled phases in different ways. As a measure of the transport efficiency, the ratio of vehicle takeoff mass to payload mass, the so-called growth factor, has been chosen. The performance assessment also includes a sensitivity analysis considering the most important design parameters: the overall efficiency of the airbreathing engines, the specific impulse of the rockets, the thrust-to-drag ratio, and the Mach number at the separation or the switch to a rocket-propelled phase.

Nomenclature

b_T = thrust specific fuel consumption
D = drag
f_f = fuel mass for return flight in units of fuel consumption until separation
f_{loss} = gravity and control loss for rocket phases
f_T = vehicle mass reduction by higher fuel consumption in the transonic regime
GF = growth factor
g = gravity constant
H = heating value of the fuel
h = height above ground
I_{sp} = specific impulse of rockets
k = $\eta_t H(1 - D/T)$
L = lift
M = Mach number
m = momentary mass of the vehicle
p = static pressure
q = dynamic pressure
T = thrust
t = time
V = velocity
W = vehicle weight
z = functions of energy
γ = specific heat ratio
ϵ = energy per unit of mass
η_t = overall propulsion efficiency
ϑ = flight-path angle
μ = mass fraction (in units of initial mass)
φ = constant part of empty mass (in units of payload)

Subscripts

A = airbreathing propelled phase
BO = burnout
e = empty mass
ec = constant part of empty mass
es = scalable part of empty mass
f = fuel
p = payload
R = rocket propelled phase
r = reference cases, defined in Table 1
S = separation
Sw = phase switching
T = transonic section
x = momentary
0 = initial
2 = second phase or second stage

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Introduction

ATTENTION throughout the world has turned to the benefits that can be gained in space transportation by combining the features of aircraft and rocket propulsion. At present, several concepts of such advanced space transportation systems are under discussion, and challenging technology programs are on the way.

A rough classification of such vehicles distinguishes single-stage-to-orbit (SSTO) and two-stage-to-orbit (TSTO) concepts. SSTO systems are propelled by airbreathing engines in a first phase and switch to rockets in a second phase, where, in principle, it is possible to overlap the operation of both types of engines. TSTO systems use, in general, airbreathing engines for propelling the first stage and rockets for the second stage. Although the SSTO systems and the first stage of TSTO systems are reusable, second stages can be reusable or expendable.

A very important common parameter for all systems is the speed, or the Mach number, at which the switching from airbreathing engines to rockets or the separation, respectively, is initiated.

Examples for the various concepts of spaceplanes are NASP, HOTOL, and SÄNGER, all horizontal takeoff concepts. Since no detailed characteristics and data of these projects are used in this study, the different basic concepts are called N type, H type, and S type, respectively. The N type corresponds to an SSTO vehicle using airbreathing engines for a Mach range until 15 and higher. The H type, also SSTO, is propelled by airbreathing engines only until about $M = 5$, and then switches to rockets. The S type is a TSTO concept with an airbreathing first stage and a rocket-propelled second stage, where the separation is assumed to be initiated at about $M = 7$.

To compare different concepts of aircraft or space transportation vehicles, a suitable figure of merit has to be applied. For transport aircraft, the distances covered in cruising flight can be used as a measure of transport effectiveness. The Breguet range equation[1] combines the chemical, propulsive, aerodynamic, and structural efficiencies in a simple analytical relation for calculating aircraft ranges. Kuechemann and Weber[2] derived from the Breguet relation the payload fraction as a measure of transport efficiency for comparing various types of subsonic and supersonic aircraft. Jones and Donaldson[3] presented an analytical relation that gives the fuel mass fraction of an airbreathing spaceplane as a function of subsystem efficiencies.

For comparing different spacecraft concepts, the ratio of vehicle takeoff mass to payload mass, the so-called growth factor (*GF*), seems to be a very suitable measure of transport efficiency. Since often the payload is fixed by mission requirements, the *GF* is a measure of size and cost of spaceplanes. In this paper a comparison of the *GF* is presented for SSTO and TSTO concepts using an analytical procedure that is based on simplified aerodynamic and engine models. The nonlinear characteristic of the *GF* also permits the judgment of the design risk. Even if the absolute values of the *GF* are debatable, the relative statements made in this study seem reliable and suitable to show some basic influences.

Equations of Motion and Solution Approach

SSTO Spaceplanes (All Airbreathing)

The longitudinal velocity V of a flight vehicle is given by the equation of motion

$$m\frac{dV}{dt} = T - D - mg\sin\vartheta \tag{1}$$

assuming T acts along the flight path. If we introduce the energy per unit mass

$$\epsilon = V^2/2 + gh \tag{2}$$

multiply Eq. (1) by V, and consider that

$$\frac{dh}{dt} = V\sin\vartheta \tag{3}$$

we obtain the relationship

$$m\frac{d\epsilon}{dt} = TV(1 - D/T) \tag{4}$$

For an airbreathing engine, the fuel mass flow is approximately proportional to the thrust

$$\frac{dm}{dt} = -b_T T \tag{5}$$

It is useful to introduce the overall efficiency of the propulsion system that is given by

$$\eta_t \frac{TV}{b_T TH} \tag{6}$$

where H is the heating value of the fuel.

By eliminating the specific fuel consumption b_T, Eq. (5) becomes

$$\frac{dm}{dt} = -\frac{TV}{\eta_t H} \tag{7}$$

Combining Eqs. (4) and (7), we obtain

$$m\frac{d\epsilon}{dm} = -\eta_t H(1 - D/T) \tag{8}$$

or

$$\int_{m_0}^{m_x}\frac{dm}{m} = -\int_{\epsilon_0}^{\epsilon_x}\frac{d\epsilon}{\eta_t H(1 - D/T)} \tag{9}$$

If the parameters η_t, H, and D/T are considered to be constant, as also assumed by Jones and Donaldson,[3] the integration in Eq. (9) can easily be carried out and provides a relationship between the momentary values of vehicle mass and energy given by

$$\frac{m_x}{m_0} = \exp\left[-\frac{\epsilon_x - \epsilon_0}{\eta_t H(1 - D/T)}\right] \tag{10}$$

This relationship is of the form

$$\frac{m_x}{m_0} = e^{-z_A} \tag{11}$$

with

$$z_A(\epsilon_0, \epsilon_x) = \frac{\epsilon_x - \epsilon_0}{\eta_t H(1 - D/T)} \equiv \frac{\Delta\epsilon_x}{k} \tag{12a}$$

The ratio D/T can be written as a function of lift-to-drag ratio L/D and thrust loading T/W

$$z_A = \frac{\epsilon_x - \epsilon_0}{\eta_t H[1 - 1/(L/D \cdot T/W)]} \tag{12b}$$

In principle, a variation of the parameters could be covered by the integration in Eq. (9) if we approximate the variables by sectionwise constant values. Then z_A would be given as a sum of terms

$$z_A = \sum_{i=1}^{n}\frac{\Delta\epsilon_i}{k_i} \tag{12c}$$

given by the energy increment $\Delta\epsilon_i$ in section i and different denominators k_i

$$k_i = \eta_t H\,(1 - D/T)|_{\text{section } i}$$

In a plot with log scaling of m_x/m_0, lines with different slopes would be obtained. The refinement of Eq. (12c) would be particularly worthwhile in the transonic regime because drag rise effect and reduction in thrust give lower values of lift-to-drag ratio L/D and thrust loading T/W, thus leading to reduced values of m_x/m_0. Equation (12c) could also be used to consider Mach effects on T/D and η_t.

The fuel mass consumed during a flight that yields the energy ϵ_x is given by

$$m_f = m_x - m_0 \tag{13}$$

so that the fuel mass fraction (ratio of fuel mass to vehicle takeoff mass) can be expressed by

$$\mu_f = \frac{m_f}{m_0} = 1 - e^{-z_A} \tag{14}$$

The mass division between empty mass, fuel, and payload can be written in the form

$$\mu_e + \mu_f + \mu_p = 1 \tag{15}$$

The empty mass of the vehicle should be split into a constant part and a part that is scaled with the takeoff mass

$$\mu_e = \mu_{ec} + \mu_{es} \tag{16}$$

If we assume that the payload has a given constant value, the constant part μ_{ec} can be specified proportional to the payload:

$$\mu_{ec} = \varphi\mu_p \tag{17}$$

Then we obtain from Eqs. (14–17) with $z_1 \equiv z_A(0, \epsilon_{BO})$ a payload fraction

$$\mu_p = \frac{e^{-z_1} - \mu_{es}}{(1 + \varphi)} \tag{18}$$

or, as the inverse, the growth factor

$$GF = \frac{m_0}{m_p} = \frac{(1 + \varphi)}{e^{-z_1} - \mu_{es}} \tag{19}$$

Figure 1 illustrates the typical trend of the growth factor. It shows the *GF* for an SSTO airbreather as a function of the scalable empty mass fraction for values of $\varphi = 1$ and $\epsilon_{BO} = 31.6$ MJ/kg. Noteworthy is the nonlinear shape of this function that ends up with a singularity that means that a payload can be brought to orbit only with infinite takeoff mass. In other words, an increase of takeoff mass is more and more absorbed by the growing structural, subsystem, and fuel masses. A 1% payload fraction ($GF = 100$) would require a scalable empty mass of not more than 29.5%.

SSTO Spaceplanes with Airbreathing and Rocket Phase (H Type and N Type)

With these types of vehicles, the first phase is propelled by airbreathing engines whereas a second phase, up to orbital conditions, relies on rocket engines. The phase switching should occur at a Mach number M_{Sw}. The corresponding height above ground can be calculated if the dynamic pressure q_S is given. Then, the relationship

$$q_{Sw} = (\gamma/2)p_{Sw}M_{Sw}^2 \tag{20}$$

can be used to determine the static pressure p_{Sw} of the atmosphere at phase switching. Because the static pressure is a known function of height $p(h)$, the height at phase transfer is also fixed. If we use, for example, the approximation

$$p = p_r \exp(-ah) \tag{21}$$

(with $p_r = 127{,}350$ N/m^2 and $a = 0.1568$ km^{-1}), it follows from Eqs. (20) and (21) that

$$h_{Sw} = \frac{1}{a} \ln \left[\frac{\gamma p_r M_{Sw}^2}{2q_{Sw}}\right] \tag{22}$$

With known values of height and Mach number, the energy at phase switching ϵ_{Sw} can also be evaluated from Eq. (2). Figure 2 gives a plot of ϵ and h vs M for a standard atmosphere if the vehicle climbs with a dynamic pressure of 50 kPa.

Then $z_{11} \equiv z_{Sw} = z_A(0, \epsilon_{Sw})$ can be determined from Eq. (12), and the mass at the beginning of phase 2 follows from

$$\frac{m_{Sw}}{m_0} = e^{-z_{11}} \tag{23}$$

In the second phase, the vehicle is driven by rocket engines. While Eq. (4) is still valid, instead of Eq. (5) the relationship

$$T = -\dot{m}gI_{sp} \tag{24}$$

must be used, or

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{T}{gI_{sp}} \tag{25}$$

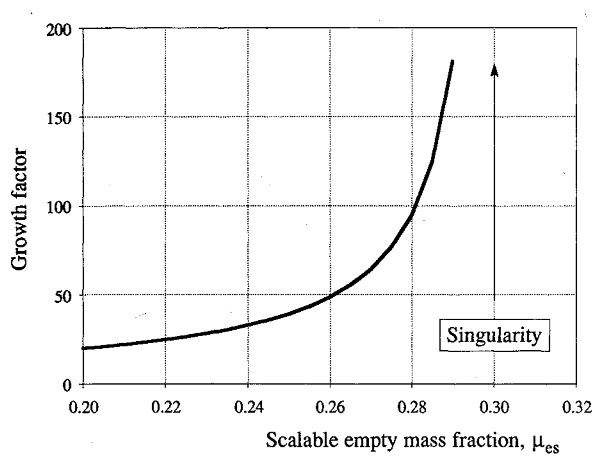

Fig. 1 Principal trend of the growth factor (*GF*).

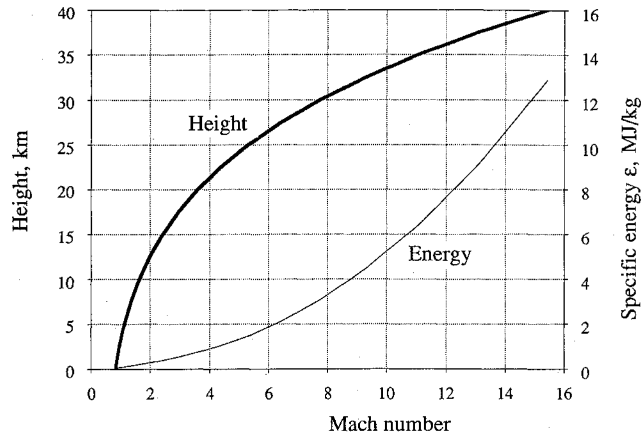

Fig. 2 Height and energy along a flight path of q = 50 kPa.

The combination of Eqs. (4) and (25) leads to

$$m\frac{\mathrm{d}\epsilon}{\mathrm{d}m} = -VgI_{sp}(1 - D/T) \tag{26}$$

Because of Eq. (2) we have

$$V = \sqrt{2(\epsilon - gh)} = \sqrt{2\epsilon}\sqrt{1 - gh/\epsilon} \tag{27}$$

Since $gh/\epsilon \ll 1$, the second root at the right hand side can be neglected so that we obtain

$$m\frac{\mathrm{d}\epsilon}{\mathrm{d}m} = -\sqrt{2\epsilon}gI_{sp}(1 - D/T) \tag{28}$$

instead of Eq. (26). The term in parentheses is considered in the form of a loss factor f_{loss} that also takes into account the loss due to turning the thrust vector against the flight direction for compensating the gravity force before getting orbital. Then Eq. (28) is written as

$$\frac{\mathrm{d}m}{m} = -(1 + f_{loss})\frac{\mathrm{d}\epsilon}{\sqrt{2\epsilon}gI_{sp}} \tag{29}$$

Corresponding to Eq. (11), we obtain

$$\frac{m_x}{m_{2,0}} = e^{-z_R} \tag{30}$$

where the initial mass at the second phase is equal to the mass at phase switching, $m_{2,0} = m_{Sw}$, and

$$z_R = \sqrt{2}(1 + f_{loss})\frac{\sqrt{\epsilon_x} - \sqrt{\epsilon_{Sw}}}{gI_{sp}} \tag{31}$$

The mass ratio at burnout of the rocket phase can be calculated from Eq. (29) with $z_{12} \equiv z_R(\epsilon_{Sw}, \epsilon_{BO})$ given by

$$z_{12} = \sqrt{2}(1 + f_{loss})\frac{\sqrt{\epsilon_{BO}} - \sqrt{\epsilon_{Sw}}}{gI_{sp}} \tag{32}$$

The growth factor *GF* of the two-phase vehicle can be determined, analogous to Eq. (19), from

$$GF = \frac{m_0}{m_p} = \frac{(1 + \varphi)}{e^{-z_{11}}e^{-z_{12}} - \mu_{es}} \tag{33}$$

TSTO Vehicles with Airbreathing/Rocket Stages (S Type)

The performance of the first and the second stages can be calculated using the relationships given in the preceding sections. For the rocket-propelled second stage, we have to consider the mass reduction at separation. The initial mass of the second stage, $m_{2,0}$, is given as the mass at separation reduced by the empty masses of the first stage and the remaining fuel mass, required as reserve and for the return flight:

$$m_{2,0} = m_{1S} - m_{ec1} - m_{es1} - f_f(m_0 - m_{1s}) \tag{34}$$

The remaining fuel mass is given as part f_f of fuel burned until separation.

With $z_{21} \equiv z_A(0, \epsilon_S)$, Eq. (34) becomes

$$m_{2,0} = e^{-z_{21}}m_0 - \varphi_1 m_p - \mu_{es1} m_0 - f_f(m_0 - e^{-z_{21}}m_0) \tag{35}$$

For the rocket-propelled stage, Eq. (31) is applied for calculating the mass fraction at burnout to the initial mass of the second stage

$$\frac{m_{2,BO}}{m_{2,0}} = e^{-z_{22}} \tag{36}$$

where $z_{22} \equiv z_R(\epsilon_S, \epsilon_{BO})$.

If the empty mass of the second stage is also split into a constant part and a part that is scaled with the initial mass of the second stage, the payload of the TSTO transport system is given as

$$m_p = m_{2,BO} - m_{ec2} - m_{es2} = e^{-z_{22}}m_{2,0} - \varphi_2 m_p - \mu_{es2} m_{2,0} \tag{37}$$

Using Eqs. (35) and (37), the overall growth factor can finally be written in the form

$$GF = \frac{1 + \varphi_2 + \varphi_1(e^{-z_{22}} - \mu_{es2})}{[e^{-z_{21}} - \mu_{es1} - f_f(1 - e^{-z_{21}})](e^{-z_{22}} - \mu_{es2})} \tag{38}$$

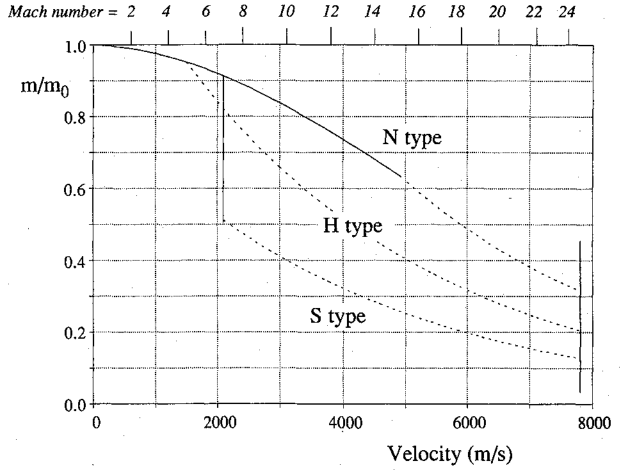

Fig. 3 Mass functions for the three types of spaceplanes.

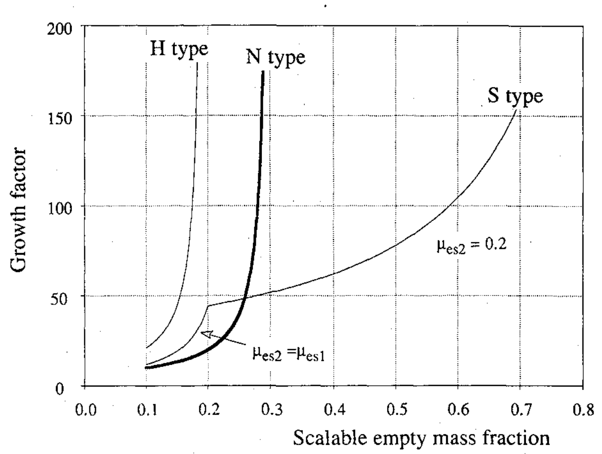

Fig. 4 *GF* for the three types of spaceplanes.

Table 1 Parameters of the reference cases

Heating value of the fuel for the airbreathing engines (LH_2), MJ/kg	H	118
Average overall propulsion efficiency of the airbreathing engines	η_t	0.4
Specific impulse of rockets, s	I_{sp}	450
Average lift-drag ratio	L/D	3
Average thrust loading	T/W	0.8
Constant part of empty mass (in units of payload) for SSTO and first stage of TSTO	φ_1	1
Constant part of empty mass (in units of payload) for second stage of TSTO	φ_2	0.2
Scalable part of empty mass fraction (first stage)	μ_{es1}	0.4
Scalable part of empty mass fraction (second stage)	μ_{es2}	0.2
Fuel mass for return flight in units of fuel consumption until separation	f_f	0.2
Gravity and control loss for rocket phases	f_{loss}	0.1
Energy at burnout, MJ/kg	ϵ_{BO}	31.8
Mach number at transition to rocket phase		
H type	M_{Sw}	5
N type	M_{Sw}	15
Mach number at separation for S type	M_S	7

Table 2 Fuel consumption

	Fuel fraction	
Concept	Airbreathing phase	Total
H type	0.048	0.796
N type	0.362	0.685
S type	0.088	0.472

Results

The data used in this study are summarized in Table 1.

In Fig. 3 the three spaceplane concepts are compared with regard to the vehicle masses as a function of velocity. The diagram is based on the same engine performance and lift/drag ratios for all three types. The figure also presents the different phase changes and stage separations. The values of fuel consumption for the airbreathing phase and the whole mission can be derived from this diagram and are given in Table 2. The S type has the lowest total fuel consumption, whereas the H type requires the highest value. The payload for the three concepts is found from the difference between burnout and empty masses. An empty mass of about 31% would bring the payload down to zero for the N type, and the corresponding value for the H type is, at 20%, still much lower. The TSTO S type makes much fewer demands on empty mass. Even if the empty mass fraction of the first stage exceeds 40% (indicated by the step in the mass curve at separation), the mass ratio of the second stage must be larger than 25% before the payload vanishes.

Figure 4 compares the three concepts with regard to the *GF* as a function of the scalable empty mass fractions. For the S type only the empty mass of the first stage is varied, while the empty mass fraction of the second stage is kept constant at $\mu_{es2} = 0.2$. In the most interesting range of 50 to 100, the *GF* is near to a singularity and highly sensitive to changes in empty mass fractions for SSTO concepts. The figure illustrates the favorable shift of the *GF* singularity for the S type and the low sensitivity even at empty mass fractions between 40 and 50%.

The more favorable *GF* of the N type compared with the H type, shown in Fig. 4, is due to the difference in the Mach number at switching from airbreathing to rocket-driven phase. The influence of this Mach number is given in Fig. 5. An increase of the switching Mach number from $M = 5$ to 10 alleviates the demands on empty mass by about 35%. But the advantage of increasing the switching Mach number is reduced by the higher demands in thermal protection and propulsion system mass.

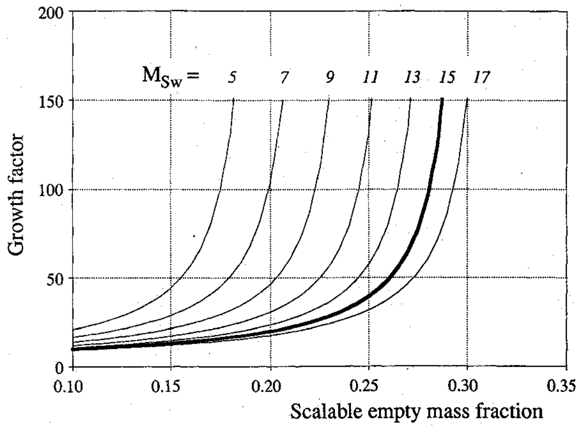

Fig. 5 Influence of Mach number at phase switching on *GF*.

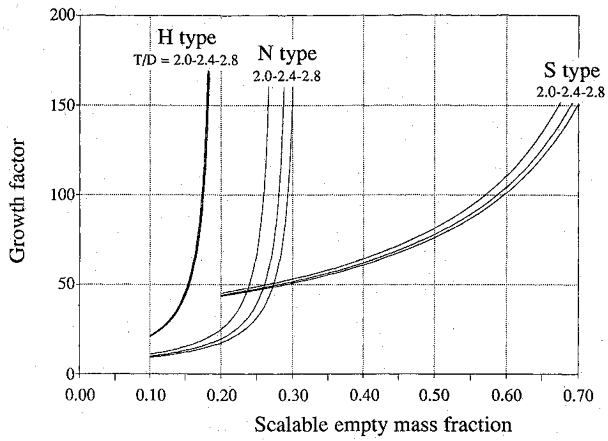

Fig. 6 Influence of $T/D = T/W \cdot L/D$ on *GF*.

The last figures are based on the reference case, where the parameter $T/D = (L/D)(T/W)$ has the value $T/D = 2.4$. The influence of this parameter on the *GF* is shown in Fig. 6 for the three concepts. Without changing the basic behavior of the *GF* curves, the singularity is shifted to lower values of the scalable empty weight if T/D is reduced. The influence depends on the Mach number regime of the airbreathing phase. Thus, in the range of $T/D = 2.0$ to 2.8, the influence is largest for the N type. A reduction of the thrust-to-drag ratio by 30% (from $T/D = 2.8$ to 2.0) would enlarge the empty mass demand by about 12%. The influence of T/D is almost negligible for the H type.

Following the context to Eqs. (12), the basic influence of a transonic regime on the *GF* can be obtained for the various concepts by multiplying e^{-z_1}, $e^{-z_{11}}$, and $e^{-z_{21}}$ in Eqs. (19), (33), and (38), respectively, by a factor $f_T < 1$. If a transonic section is embedded in a flight trajectory, which is otherwise described by the reference parameters given in Table 1, z_A is given by

$$z_A = z_{Ar} + (1/k_T - 1/k_r)\Delta\epsilon_T \tag{39}$$

with z_{Ar} denoting the value of the reference case. Then f_T can be written in the form

$$f_T = \exp[-(1/k_T - 1/k_r)\Delta\epsilon_T] \approx \exp[-(1/k_T)\Delta\epsilon_T] \tag{40}$$

and may be approximated by

$$f_T \approx \exp\left\{-\frac{(T/D)_T}{\eta_{tT} H[(T/D)_T - 1]}\Delta\epsilon_T\right\} \tag{41}$$

which is independent from the reference case. If the transonic section is specified as extending from $M = 0.8$ to 1.8 and, furthermore, values of $\eta_{tT} = 0.2$ and $(T/D)_T = 1.1$ are assumed, a value of $f_T = 0.95$ is obtained. Figure 7 compares the influence of the transonic section for the N and S type concepts. For both concepts, the singularities of the *GF* are considerably reduced, with the S type being more sensitive.

For the TSTO vehicles the significant design parameter is the empty mass of the second stage, as visualized in Figs. 8 and 9. The *GF* can be considerably influenced by the Mach number at stage separation and the specific impulse of the rocket stage. As shown in Fig. 8, a decrease of the separation Mach

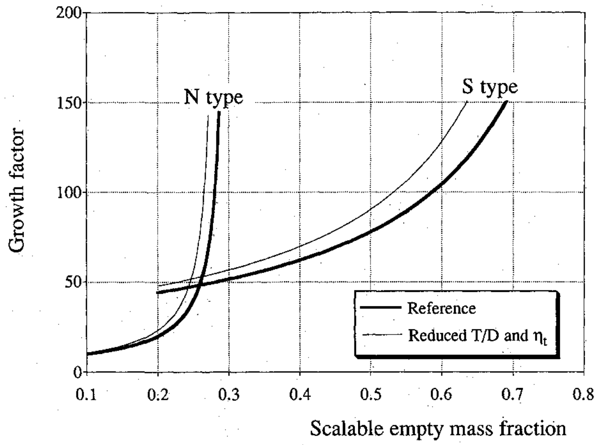

Fig. 7 Influence of the higher fuel consumption in the transonic regime on *GF* for the N type and S type ($f_T = 0.9$).

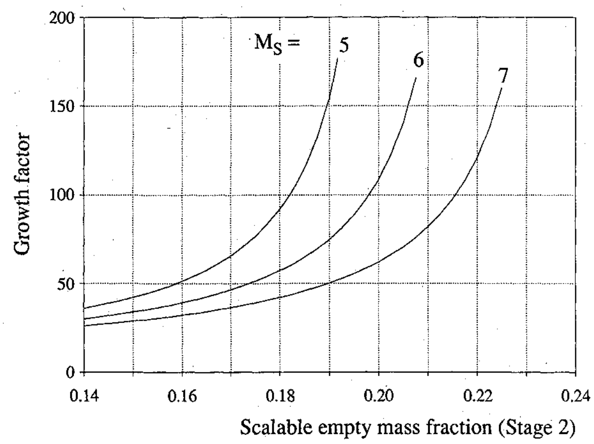

Fig. 8 Influence of the Mach number at separation on *GF* for the S type.

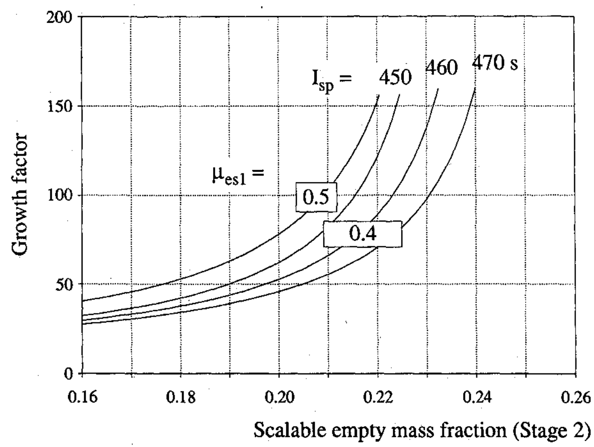

Fig. 9 Influence of specific impulse on *GF* for the S type.

number from $M_s = 7$ to 5 would shift scalable empty mass, required for a given GF, to lower values by about 20%.

The influence of the specific impulse of the rocket stage (S type) is shown in Fig. 9. An increase of I_{sp} by 10 s would increase the required scalable empty weight favorably by 3.5%. The reduction of the empty weight of the first stage by 20% (from $\mu_{es1} = 0.5$ to 0.4) has a similar effect.

Conclusions

Analytical solutions of the equations of motion, which are based on simplified aerodynamic and engine models, permits the performance comparison of space vehicles with airbreathing and rocket-propelled phases. The growth factor (*GF*) has been chosen as a suitable measure for this performance assessment. Even if the absolute values of the *GF* are debatable, the relative statements seem reliable and suitable to show some basic influences. The significant results of this study are the following.

1) The empty mass fractions of SSTO and of the first stage of TSTO vehicles required for a chosen *GF* differ considerably in that TSTO systems are much less demanding.

2) In the most interesting range of 50 to 100, the *GF* is near to a singularity and highly sensitive to changes in empty mass fractions for SSTO concepts whereas the first stage of TSTO vehicles is much less sensitive.

3) A significant parameter for SSTO spaceplanes is the Mach number at which the transition from airbreathing to rocket engines is initiated. An increase of the switching Mach number from $M = 5$ to 10 alleviates the demands on empty mass by about 35%.

4) The thrust/drag ratio (which can be factored into lift/drag ratio and thrust loading) is shown to be a major influence for the N type. A reduction of the thrust-to-drag ratio by 30% (from $T/D = 2.8$ to 2.0) would enlarge the empty mass demand by about 12%.

5) The higher fuel consumption required for penetrating the transonic regime, because of the reduced thrust/drag ratio, is of considerable influence on the *GF*. TSTO spaceplanes are more sensitive to transonic effects than SSTO.

6) For TSTO vehicles the significant design parameter is the empty mass fraction of the second stage. Increasing the separation Mach number and the specific impulse of the rocket engines is favorable for reducing the *GF* and its sensitivity to changes of the empty mass.

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Associate Editor